Mass Transfer in Evaporating Falling Liquid Film Mixtures

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Falling liquid films are encountered in several processes for heat- and mass-transfer applications, particularly for low-pressure evaporation. Because of a continuous vapor space, the pressure drop associated with falling film evaporation is very low compared to a pool or convective boiler. Falling liquid film systems are suitable for processing viscous and heat-sensitive fluids, and are inherently efficient thermodynamically since high heat flux can be obtained with a low-temperature driving force. In view of this commercial importance, it is surprising that fundamental analyses of the basic phenomena are quite incomplete.

One basic problem is to describe axial and transverse concentration gradients in a two-component falling liquid film resulting from preferential evaporation of the more volatile component and diffusive resistance in the liquid film. The related process of evaporation of liquid mixtures in upward annular flow has been examined by Shock (1976). Condensation and evaporation in wetted-wall columns were studied by Davis et al. (1984) for the special case where the resistance to mass transfer in the liquid phase is taken to be negligible. More recently, Palen (1988) and Palen and Chen (1989) found in an experimental investigation that liquid-side mass-transfer resistance can be significant for falling films, causing substantial reductions in the effective evaporative heat-transfer coefficient.

This work provides the formulation of the governing equations and boundary conditions that describe the evaporation of two-component liquid films falling down a vertical surface. Based on these governing equations, the concentration profile and axially developing Sherwood numbers are calculated by a numerical procedure. An analytical representation of the asymptotic Sherwood number with a value of 140/33 is derived, and the results of the numerical calculation are shown to approach this analytical solution at long lengths.

The analogous heat-transfer problem has been studied previously. Yih and Huang (1980) and Faghri (1976), as reported by Yih (1986), give Nusselt numbers for the cases of constant wall temperature and uniform heat flux. The solution for the Sherwood number derived in this work differs from the heat-transfer problems described by these studies in one aspect. For the heat-transfer problem, the applied temperature gradient is at the wall where the fluid velocity is zero. In the mass-transfer problem, the applied concentration gradient is at the liquid-vapor interface where the velocity maintains its maximum value.

Governing Equation Development

In developing general governing equations and boundary conditions for the concentration profile of a two-component falling liquid film, Bose and Palmer (1984) suggested the use of mass rather than mole fractions since the total mass density of the liquids is less sensitive to changes in concentration than molar density. In this work, the following simplifying assumptions are made: (1) steady state; (2) constant fluid properties; (3) the film is laminar and wave effects are ignored; (4) the film enters at saturation conditions; (5) heat lost or gained by the film due to the axial change in saturation temperature is negligible compared to the latent heat, and sensible heating is negligible; (6) heat is transferred through surface evaporation, and no nucleate boiling is present; (7) the film thickness is smaller than the tube diameter so that the flat plate approximation is appropriate; (8) circumferential symmetry exists; (9) vapor velocities are low (laminar flow) and vapor shear effects are negligible; and (10) surface tension forces in the axial direction are smaller than the gravity force.

Coordinates are defined in Figure 1, which is a schematic of a film flowing down a heated vertical surface. A parabolic velocity profile is shown along with a concentration profile for the more volatile component in the mixture.

Scale analysis

Governing equations are derived from the continuity, momentum, component continuity and energy equations. Consistent with assumption 7, descriptive equations for thin-film flow on a tube are written in Cartesian coordinates with terms $O(h_o/R_o)$ neglected. In the main flow direction, aligned with the gravitational force, the velocity component is designated by u and the velocity perpendicular to the tube is v.

Scaled variables are defined by:

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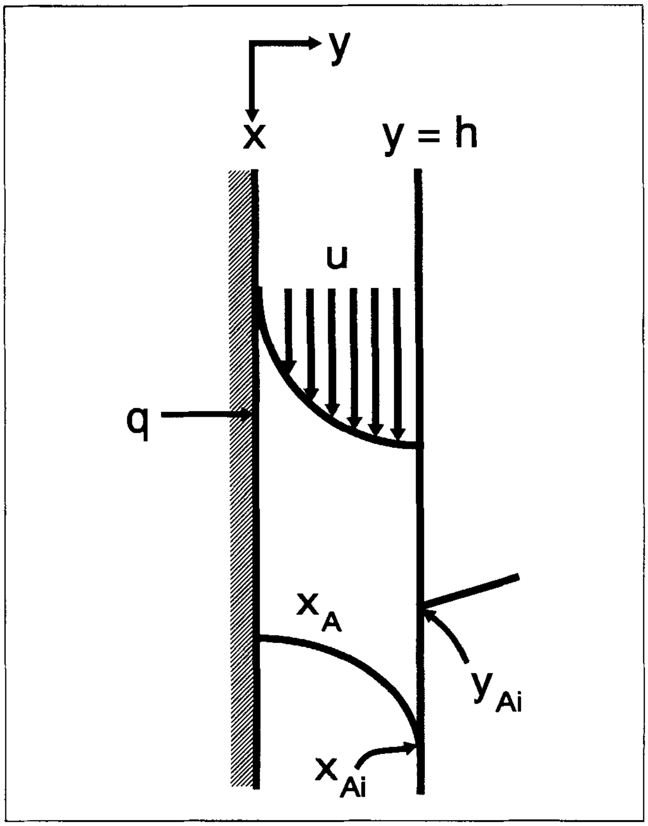

Figure 1. Concentration and velocity profiles in a falling liquid film.

$$u' = \bar{u}u,\quad v' = \frac{h_o}{L}\bar{u}v,\quad x' = Lx,\quad y' = h_o y,\quad h' = h_o h,$$

$$\text{and } p' = \mu\frac{\bar{u}}{h_o}p, \tag{1}$$

where $\bar{u} = \rho g h_o^2/(3\mu)$ is the average x-direction velocity as defined using Nusselt theory, and the initial inlet film thickness, $h_o = \sqrt[3]{3\Gamma\mu/(\rho^2 g)}$, is also defined using Nusselt theory. Primed variables represent the dimensional form and are defined in the Notation section. Here, the pressure is scaled with a viscous pressure. Unprimed dimensionless variables u, v, x, y, p, and h are all taken to be $O(1)$ quantities.

As a result of the scaling, the nondimensional groups that emerge are:

$$Fr^2 = \frac{\bar{u}^2}{g h_o},\quad Re = \frac{\rho\bar{u}h_o}{\mu},\quad \text{and } Sc = \frac{\mu}{\rho\mathfrak{D}_{AB}}. \tag{2}$$

In terms of the nondimensional quantities, the continuity, momentum and component continuity equations, neglecting terms of $O(h_o/R_o)$, $O(h_o/L)$ and smaller, become:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3}$$

$$\frac{\partial^2 u}{\partial y^2} = -\frac{Re}{Fr^2} = -3,\quad \frac{\partial p}{\partial y} = 0, \tag{4}$$

$$\delta\left(u\frac{\partial x_A'}{\partial x} + v\frac{\partial x_A'}{\partial y}\right) = \frac{\partial^2 x_A'}{\partial y^2} \tag{5}$$

where

$$\delta = Re\,Sc\left(\frac{h_o}{L}\right).$$

Davis et al. (1984) and Baumann and Thiele (1990) study the special case where δ is small and conclude that the mass-transfer resistance in the liquid phase is negligible. In this study, the lefthand side of Eq. 5 is retained, since δ is typically $O(1)$. Nonnegligible mass-transfer resistance in the liquid phase is consistent with the experiments described by Palen and Chen (1989).

Burelbach et al. (1988) give the kinematic boundary condition for an interface that is not a material surface. Consistent with the scaling (Eq. 1) and neglecting terms $O(h_o/R_o)$ and smaller, we have at the interface ($y = h$),

$$J = v(h) - u(h)\frac{\partial h}{\partial x} \tag{6}$$

where $J = (qL)/(\lambda_{mix}\rho h_o \bar{u})$. Here, $u(x,h) \equiv u(h)$ and the additional dependence on x is to be understood. The mixture latent heat, $\lambda_{mix} = y_{Ai}'(\lambda_A - \lambda_B) + \lambda_B$. Since the mixture latent heat changes with the interface concentration, J is not a constant.

Velocity profile

The velocity profile is obtained from Eq. 4 and appropriate boundary conditions. At the wall, $y = 0$, the no-slip boundary condition:

$$u = v = 0 \tag{7}$$

is applied. General boundary conditions at the film surface, $y = h$, including surface tension phenomena, are given by Levich and Krylov (1969). These general boundary conditions are based on a force balance at the interface. However, for the experimental conditions described by Hoke (1991) using aqueous mixtures of ethylene glycol and propylene glycol, the surface tension effect in the axial direction is small, and the approximation of zero shear at the interface,

$$\frac{\partial u}{\partial y} = 0, \tag{8}$$

is appropriate. The error introduced to the bulk velocity by neglecting the shear caused by surface tension is calculated to be less than 0.5% for these experiments. Details of the error analysis are given by Hoke.

With these boundary conditions, the velocity profile, u, for steady uniform laminar flow of a smooth film on a vertical plane is

$$u = 3yh - \frac{3}{2}y^2 \tag{9}$$

and was derived first by Nusselt (1910).

The normal component of the film velocity, v, derived using the continuity equation (Eq. 3), the no-slip boundary condition (Eq. 7), and Eq. 9 is:

$$v = -\frac{3}{2} y^2 \frac{dh}{dx} . \tag{10}$$

Film thickness profile

The liquid film thickness decreases axially due to the evaporation of both components of the film. Substituting Eqs. 9 and 10 evaluated at the interface, $y=h$, into the kinematic boundary condition (Eq. 6) gives:

$$J = -3h^2 \frac{dh}{dx} . \tag{11}$$

Integrating Eq. 11 with the boundary condition $h=1$ at $x=0$ gives:

$$h = \left(1 - \int_o^x J\, dx\right)^{1/3} \tag{12}$$

for the film thickness as a function of axial position.

Concentration profile

The dominant approximation of the continuity equation for the more volatile component, A, is given by Eq. 5. This equation indicates that A moves down the tube primarily due to convective flow and that both convective and diffusive flow are important for transport through an evaporating film. Three boundary conditions are required if a solution to Eq. 5 is to be obtained. At the tube inlet, $x=0$, the boundary condition is:

$$x_A' = x_{Ao}' \tag{13}$$

where x_{Ao}' is the inlet mass fraction of component A. Since there is no mass flux through the wall,

$$\frac{\partial x_A'}{\partial y} = 0 \tag{14}$$

at $y=0$. A mass flux is imposed at the interface due to the preferential evaporation of component A. This boundary condition at the liquid-vapor interface, $y=h$, is given by:

$$\frac{\partial x_A'}{\partial y} = \frac{q(x_{Ai}' - y_{Ai}')h_o}{\rho \mathfrak{D}_{AB}[y_{Ai}'(\lambda_A - \lambda_B) + \lambda_B]} \equiv \Lambda' \tag{15}$$

and is derived below.

An energy balance, neglecting sensible heat effects, requires:

$$q = j_{Ai}'\lambda_A + j_{Bi}'\lambda_B . \tag{16}$$

Mass transfer to the vapor can be described by:

$$j_{Ai}' - y_{Ai}'(j_{Ai}' + j_{Bi}') = \alpha_m^{\bullet}(y_{Ai}' - y_{Ab}') \tag{17}$$

where a mass-transfer coefficient $\alpha_m^{\bullet}$ is used to describe the diffusive contribution to mass transfer in the vapor. A scoping analysis where the analogy between heat (Leveque solution for laminar flow) and mass transfer is used for $\alpha_m^{\bullet}$ indicates that $\alpha_m^{\bullet}(y_{Ai}' - y_{Ab}')$ is negligible in comparison to the convective terms. This suggests that the mass flux of A from the interface due to evaporation is much larger than that diffusing back to the surface. Note that rate factors from the film theory used to correct for the effect of mass transfer on the transfer coefficients, described by Bird et al. (1960), are positive and large, and thus decrease significantly the magnitude of the mass-transfer coefficient $\alpha_m^{\bullet}$. Equation 17 reduces to:

$$j_{Ai}' - y_{Ai}'(j_{Ai}' + j_{Bi}') = 0. \tag{18}$$

On the liquid side of the liquid-vapor interface, the boundary condition is:

$$j_{Ai}' = -\frac{\rho \mathfrak{D}_{AB}}{h_o} \frac{\partial x_A'}{\partial y} \Big|_{y=h} + x_{Ai}(j_{Ai}' + j_{Bi}') \tag{19}$$

as given in equivalent form by Bose and Palmer (1984) and Ruckenstein et al. (1983). Writing Eqs. 16, 18 and 19 in matrix form and solving for

$$\frac{\partial x_A'}{\partial y} \Big|_{y=h}$$

using Cramer's rule gives Eq. 15.

This imposed mass flux depends on the interfacial composition, x_{Ai}', which is not known *a priori*, but must be found as part of the solution. Fortunately, the surface composition varies only slightly for low to moderate heat fluxes as the film flows down the tube, and thus the mass flux is approximately constant. Vapor liquid equilibrium gives y_{Ai}' as a function of x_{Ai}'.

An analytical solution for the concentration profile does not exist since boundary condition (Eq. 15) depends on the developing solution, and therefore a computational solution is required. In addition to calculating concentration values, the axially developing Sherwood number may be calculated numerically and compared to the analytical solution of the asymptotic Sherwood number for an evaporating binary liquid falling film.

Asymptotic Sherwood Number

The Sherwood number at large axial distances ($x \to \infty$) for a purely laminar evaporating binary liquid film can be derived analytically for the special case where film thickness changes due to evaporation are negligible ($J \to 0$). Consequences of this limitation are $h=1$ and $v=0$. As noted above, this problem differs from the corresponding heat-transfer problem for falling films.

For the fully developed concentration profile case, a mass balance on the light component is:

$$\frac{\partial x_A'}{\partial x} \int_0^y u\, dy + j_A = 0 \tag{20}$$

and at the interface

$$\frac{\partial x'_A}{\partial x}\int_0^1 u\,dy + j_{Ai} = 0 \qquad (21)$$

where the dimensionless diffusive mass flux is

$$j_A = -\frac{L\mathfrak{D}_{AB}}{\bar{u}h_o^2}\frac{\partial x'_A}{\partial y}\,. \qquad (22)$$

Substituting Eq. 9 into Eq. 21 and evaluating the integral gives:

$$\frac{\partial x'_A}{\partial x} = -j_{Ai}. \qquad (23)$$

Evaluating the integral in Eq. 20 gives, with substitutions from Eqs. 9, 22 and 23:

$$-j_{Ai}\left(\frac{3}{2}y^2 - \frac{1}{2}y^3\right) = \frac{L\mathfrak{D}_{AB}}{\bar{u}h_o^2}\frac{\partial x'_A}{\partial y}\,.$$

Further integration and re-arrangement provides the result:

$$x'_A = x'_{Aw} - \frac{\bar{u}h_o^2 j_{Ai}}{2L\mathfrak{D}_{AB}}\left(y^3 - \frac{y^4}{4}\right) \qquad (24)$$

where the boundary condition $x'_A = x'_{Aw}$ at $y = 0$ has been used.

The average or bulk concentration over a cross-section of the film is obtained by:

$$x'_{Ab} = \frac{\int_0^1 x'_A u\,dy}{\int_0^1 u\,dy} \qquad (25)$$

and by substituting Eqs. 9 and 24 into Eq. 25 and integrating:

$$x'_{Ab} = x'_{Aw} - \frac{39}{280}\frac{\bar{u}h_o^2 j_{Ai}}{L\,\mathfrak{D}_{AB}}\,. \qquad (26)$$

The interfacial mass fraction is by Eq. 24,

$$x'_{Ai} = x'_{Aw} - \frac{3\bar{u}\,h_o^2 j_{Ai}}{8\,L\,\mathfrak{D}_{AB}}\,. \qquad (27)$$

From Eqs. 26 and 27, and the definition of the mass-transfer coefficient,

$$j'_{Ai} = \frac{\rho\bar{u}\,h_o}{L}\,j_{Ai} = k'_L\,(x'_{Ab} - x'_{Ai})\;, \qquad (28)$$

we obtain

$$Sh \equiv \frac{k'_L h'}{\rho\mathfrak{D}_{AB}} = \frac{k'_L h_o}{\rho\mathfrak{D}_{AB}} = \frac{140}{33} \qquad (29)$$

as the asymptotic Sherwood number for large x.

As pointed out by Churchill (1991), it is interesting to compare this result with the heat-transfer case with inverted boundary conditions, $x \to \infty$ and uniform heating. Here, the Nusselt number (based on the film thickness) = (70/34) = (35/17), which is approximately half of the Sherwood number (Eq. 29).

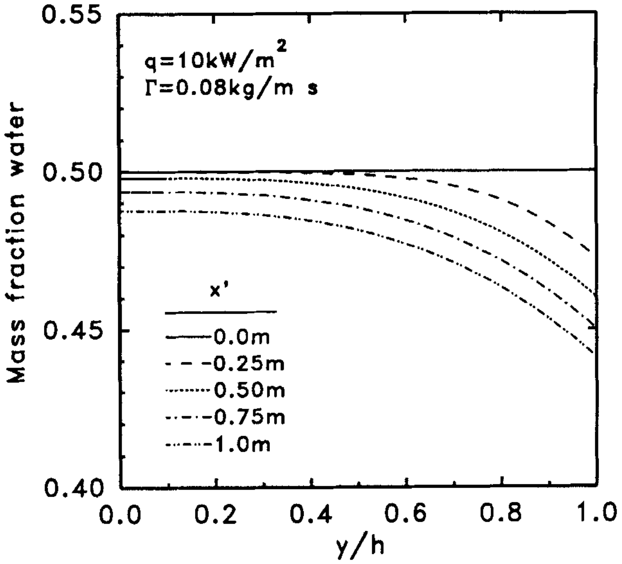

Figure 2. Calculated concentration profiles in a falling liquid film.

Numerical Solution ($J \neq 0$)

For the developing region (small x) an analytical solution to Eq. 5 with boundary conditions (Eqs. 13-15) does not exist, and thus a computational solution is required. A finite difference solution is found by backwards differencing the first derivative in x and central differencing the first and second derivatives in y. All difference forms used in the numerical computation are second-order-accurate. This discretization provides a tridiagonal matrix for the y-direction dependence which can be solved efficiently using the Thomas algorithm. In the x-direction the solution is found by a marching procedure. Updated values of h, (dh/dx), and dy are calculated at each x-position.

Figure 2 shows the calculated concentration profiles for an example system at various positions along the tube. The example system chosen is a 50 mass % ethylene glycol-water mixture with a mass-flow rate of 0.08 kg/m•s and a heat flux of 10 kW/m^2. As expected, the interface composition of the more volatile component, A, decreases as the film progresses down the vertical surface.

In addition to solving for the two-dimensional concentration profile, the Sherwood number may be calculated as a function of axial position. The developing Sherwood number values based on the "actual" mass flux (mass flux depends on interfacial concentration, see Eq. 15) for our example system are compared to values obtained for the cases of constant mass flux and constant interfacial composition. Developing Sherwood numbers for these latter cases are described next, and the effect of J on the value of the developing Sherwood number is determined.

For the case of constant interfacial mass flux, j_{Ai}, a dimensionless concentration is defined by:

$$\theta_j = \frac{\rho \mathfrak{D}_{AB}(x'_A - x'_{Ao})}{j'_{Ai} h_o}, \tag{30}$$

where x'_{Ao} is the uniform concentration at $x=0$. For a constant interface concentration

$$\theta_{x'} = \frac{x'_A - x'_{Ai}}{x'_{Ao} - x'_{Ai}} \tag{31}$$

is defined, where x'_{Ai} is the concentration at the interface at $x>0$. Introducing Eq. 30 into Eq. 5 gives:

$$\delta\left(u\frac{\partial \theta_j}{\partial x} + v\frac{\partial \theta_j}{\partial y}\right) = \frac{\partial^2 \theta_j}{\partial y^2} \tag{32}$$

and Eq. 31 into Eq. 5:

$$\delta\left(u\frac{\partial \theta_{x'}}{\partial x} + v\frac{\partial \theta_{x'}}{\partial y}\right) = \frac{\partial^2 \theta_{x'}}{\partial y^2}. \tag{33}$$

The boundary conditions for the constant flux case are:

$$\theta_j = 0\ (x=0),\ \frac{\partial \theta_j}{\partial y} = 0 (y=0),\ \text{and}\ \frac{\partial \theta_j}{\partial y} = -1\ (y=h). \tag{34}$$

For the constant x_{Ai} case, the boundary conditions are:

$$\theta_{x'} = 1\ (x=0),\ \theta_{x'} = 0\ (y=h),\ \text{and}\ \frac{\partial \theta_{x'}}{\partial y} = 0\ (y=0). \tag{35}$$

Equations 32 and 33 are solved numerically using the same differencing technique described earlier with appropriate boundary conditions to give the axial and transverse variations of θ_j and $\theta_{x'}$.

Using the definition of the Sherwood number, together with the definition of the mass-transfer coefficient, it can be shown that

$$Sh_j = \frac{h}{\theta_{j,b} - \theta_{j,i}}, \tag{36}$$

for the constant flux case where the dimensionless bulk concentration,

$$\theta_{j,b} = \frac{\int_0^h \theta_j\, u\, dy}{\int_0^h u\, dy},$$

and the interface concentration,

$$\theta_{j,i} = \theta_j (y=h).$$

Similarly, it can be shown that

$$Sh_{x'} = -\frac{h(\partial \theta_{x'}/\partial y)_{y=h}}{\theta_{x',b}} \tag{37}$$

for the constant x'_{Ai} case.

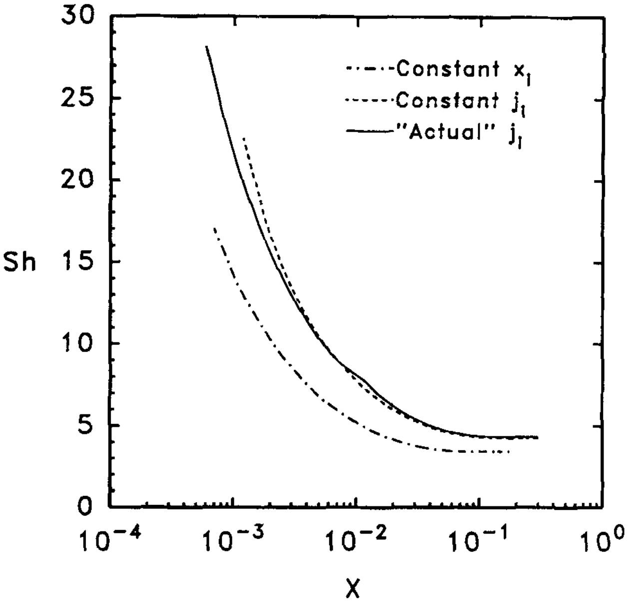

Figure 3. Calculated Sherwood number vs. dimensionless position.

Developing and asymptotic values of Sh can be calculated for the example 50 mass % ethylene glycol-water system discussed earlier and compared to the results provided by the constant interfacial mass flux and constant interface concentration cases. From Eqs. 15, 22 and 28, and the definition of the Sherwood number, Sh can be evaluated by:

$$Sh = -\frac{h\Lambda'}{x'_{Ab} - x'_{Ai}} \tag{38}$$

where x'_{Ab} is defined similarly to $\theta_{j,b}$. For this case, the interfacial composition and thus the mass flux is allowed to vary.

Calculated values of Sh are shown plotted against $X = x/\delta$ in Figure 3. For the constant j_{Ai} and x'_{Ai} cases, the values shown correspond to the special case where $J \to 0$. The effect of evaporation on the value of the developing Sherwood number is small, especially for the constant j_{Ai} case. Values were calculated and compared for cases where 30% of the film is evaporated ($J=0.3$) and values of δ varying between 0.1 and 8.0. For the constant j_{Ai} case, values of the developing Sherwood numbers are within 3.3% of the values obtained for $J \to 0$. For the constant x'_{Ai} case, values are within 12.5%. Asymptotic values of the Sherwood number for $J \to 0$ are 4.2424 and 3.4145 for the constant j_{Ai} and x'_{Ai} cases, respectively. The appropriate dimensionless development lengths are 0.116 and 0.064. These are the points at which the Sherwood number is within 1% of its asymptotic value. Note that the values for the example 50 mass % system approach those for the constant mass flux case indicating that the mass flux is approximately constant.

Palen (1988) provides Sherwood numbers which were calculated from the results of heat-transfer experiments where films of saturated ethylene glycol and water mixtures were used. The experimental heat-transfer coefficient is defined:

$$h_m \equiv \frac{q}{T_w - T_{si}} \tag{39}$$

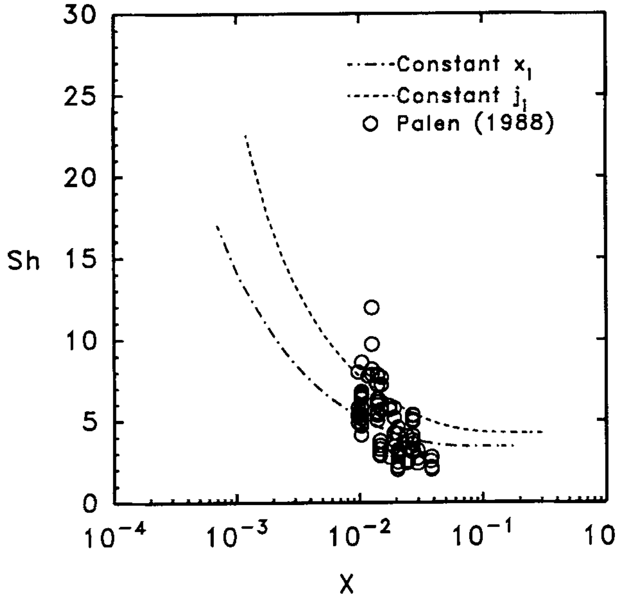

Figure 4. Comparison of experimental and theoretical Sherwood numbers.

where the wall temperature, T_w, and the heat flux, q, are measured quantities, and the saturation temperature at the interface, T_{si}, is calculated from the bulk composition. The bulk composition is determined from a differential flash calculation. The Chun-Seban (1971) correlation is recognized as a good means of calculating the heat-transfer coefficient for evaporating single-component films. Palen and Chen (1989) noted that heat-transfer coefficients calculated using the Chun-Seban correlation overpredicted their data by as much as a factor of 1.8. Attributing the difference to mass-transfer resistance and using film theory to develop a correction to the pure-component vaporization heat-transfer coefficient, a mass-transfer coefficient can be calculated. For additional details, see Palen (1988) and Palen and Chen (1989).

Sherwood numbers reported by Palen are plotted in Figure 4 along with theoretical values for constant mass flux and constant interfacial composition. While there is significant scatter in these data owing to the necessary assumptions and approximations in the calculational procedure of the mass-transfer coefficient, the data are in the appropriate range and in general agreement with the present analysis.

Conclusions

Governing equations and boundary conditions are provided for an evaporating, two-component falling liquid film driven by gravity down a vertical surface. For the case of evaporation to a laminar vapor-phase mass-transfer resistance in the liquid phase is significant, whereas the vapor phase resistance is negligible. The governing equations are solved numerically using finite differencing to provide concentration profiles and axially developing Sherwood numbers.

An analytical solution for the asymptotic Sherwood number, assuming constant mass flux at the interface, is derived. The value of the Sherwood number is roughly twice the value of the Nusselt number for a similar heat-transfer case where the boundary conditions are inverted. The Sherwood number calculated numerically for a sample case of axially varying mass flux compares favorably to the analytical asymptotic Sherwood number, indicating that the mass flux is approximately constant for low to moderate heat fluxes.

The effect of evaporation on the value of the developing Sherwood number is small, especially for the constant j_{Ai} case, since even when 30% of the film is evaporated, values differed by less than 3.3% of values for $J \rightarrow 0$.

Notation

$\mathfrak{D}_{AB}$ = diffusivity, m^2/s
Fr = Froude number, $\overline{u}^2/g\ h_o$
g = gravitational constant, 9.8 m/s^2
h = dimensionless film thickness
h' = film thickness, m
h_m = heat-transfer coefficient
h_o = inlet film thickness defined by Nusselt theory, m
j_A = dimensionless mass flux of component A in liquid
j_{Ai} = dimensionless mass flux of component A at liquid-vapor interface
j'_{Ai} = mass flux of component A at liquid-vapor interface, $kg/m^2 \bullet s$
j'_{Bi} = mass flux of component B at liquid-vapor interface, $kg/m^2 \bullet s$
J = dimensionless mass flux, $qL/(\lambda_{\text{mix}} \rho h_o \overline{u})$
k'_L = liquid mass-transfer coefficient, $kg/m^2 \bullet s$
L = heated length, m
p = dimensionless pressure
p' = pressure, Pa
q = heat flux, W/m^2
R_o = tube radius, m
Re = Reynolds number, $(\rho \overline{u} h_o)/\mu = \Gamma/\mu$
Sc = Schmidt number, $\mu/(\rho \mathfrak{D}_{AB})$
Sh = Sherwood number, $(k'h')/(\rho \mathfrak{D}_{AB})$
Sh_j = Sherwood number for constant mass flux
$Sh_{x'}$ = Sherwood number for constant interface concentration
T_{si} = saturation temperature at the interface
T_w = wall temperature
u = dimensionless velocity in x-direction
u' = x-direction velocity, m/s
$\overline{u}$ = average x-direction velocity as defined by Nusselt theory, m/s
v = dimensionless velocity in y-direction
v' = y-direction velocity, m/s
x = dimensionless x'
x' = axial coordinate, aligned with gravity, m
x'_A = liquid mass fraction component A
x'_{Ab} = bulk liquid mass fraction component A
x'_{Ai} = liquid mass fraction component A at liquid-vapor interface
x'_{Ao} = inlet liquid mass fraction component A
x'_{Aw} = liquid mass fraction component A at the wall
X = x/δ
y = dimensionless y'
y' = coordinate perpendicular to tube surface, m
y'_{Ai} = vapor mass fraction component A at liquid-vapor interface
y'_{Ab} = bulk vapor mass fraction component A

Greek letters

$\alpha^{\bullet}_m$ = vapor side mass-transfer coefficient, $kg/m^2 \bullet s$
Γ = mass-flow rate per unit wetted perimeter, $kg/m \bullet s$
δ = dimensionless parameter, $Re\ Sc\ (h_o/L)$
θ_j = dimensionless concentration, Eq. 30
$\theta_{j,b}$ = dimensionless bulk concentration
$\theta_{j,i}$ = dimensionless concentration at liquid-vapor interface
$\theta_{x'}$ = dimensionless concentration, Eq. 31

λ_A = latent heat component A, J/kg
λ_B = latent heat component B, J/kg
λ_{mix} = mixture latent heat, J/kg
Λ' = dimensionless mass concentration gradient, Eq. 15
μ = liquid viscosity, kg/m•s
ρ = liquid density, kg/m^3

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